An analytical design methodology for microelectromechanical (MEM) filters

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Received 19 January 2004; received in revised form 27 August 2004; accepted 27 August 2004

Available online 30 November 2004

Abstract

The design of coupling spring for a microelectromechanical (MEM) filter is driven by the requirements of bandwidth. A novel design aspect of the coupling spring design is presented which follows mechanical–electrical–mechanical domains in order. A designed second order MEM bandpass filter consists hereby of two laterally/vertically driven resonance structures, which are coupled by a mechanical spring. For these structures, an equivalent electrical representation can be obtained and in this paper, both the series and parallel electrical equivalent circuits are analyzed. The transfer function and the correlation between the bandwidth and the coupling spring for these filter equivalents are derived. The analysis yields a complete design procedure for MEM bandpass filters, which is presented along with simulation results. With changing design parameters the filter response has been demonstrated to be maximally flat as in the case of Butterworth-filters, or to have a response allowing a wider bandwidth (as much as double) than in the former case. Analytical studies of both the extreme responses are presented with simulation results.

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Keywords: MEMS; Bandpass filters; Coupling spring; Series/parallel RLC; Electrical equivalent

1. Introduction

Microelectromechanical (MEM) resonator based filters are considered as potential replacements of prevalent tank circuits due to their very high quality factor and the chip integration compatibility. Presently, however, these MEM filters are incompatible for the industry as compared to the prevalent components for instance SAW/crystal resonators and filters, which have obtained industrial standard due to proper insertion loss and bandwidth.

The main advantages of MEM filters are their chip integration capability as achieved by [1], as well as their reduced size, provided that they match the prevalent requirements of a high quality factor, selectivity and bandwidth.

MEM based filters are considered most suitable for reduced size implementations. Replacing SAW’s and ceramics by equivalent MEM devices will result in single chip implementations of large systems, hence taking the efforts for attaining miniaturized wireless technology, which is one of the target applications, a step further. A comparison between a typical SAW resonator and a clamped-clamped beam micromechanical resonator of comparable frequency yields a size reduction of several orders of magnitude [2].

Another attractive option for MEM filters is integrability with conventional CMOS circuitry. Present micromachining techniques allow isolated microstructures to be fabricated over a pre-processed conventional CMOS integrated circuitry, for example CMOS compatible accelerometers [3] and micro-mirrors [4]. This mergence allows for complete systems to be integrated onto a single chip, hence considerably reducing the size of the overall system like generic heterodyne receiver and its associated transmitter. The integration would employ another class of mechanical transducers for instance resonators as demonstrated in [5] and [6], oscillators in [7] and [8], and filters in [9] obtained by using surface micromachining.
MEM filters have also been demonstrated to have a very high quality factor (as high as 80,000 under vacuum [10]), which allows smaller insertion loss and better control over shape factor. Micromachined bandpass filters have been demonstrated both in high frequency (e.g. 14.5 MHz [11]) and intermediate frequency regions (e.g. 455 kHz [12]) with achieved bandwidths in the order of 0.2 and 0.09%, respectively. The insertion losses were as small as 1 dB.

Using the advantage of high quality factor, MEM based filters are implemented both perpendicular and parallel to the substrate. Whereas, for vertically driven structures, the squeeze film damping in the thin capacitive gap region is the major energy dissipation factor [1]. As far as laterally driven microstructures are concerned, the major factor affecting the quality factor is Couette air flow in the gap between the structure and the substrate [13].

These dissipation mechanisms influence the choice of structure in filter design, which is carried out under the specifications of center frequency, the bandwidth which yields the quality factor and the insertion loss. The design of MEM filters follow a cookbook approach in which the coupling spring coefficient $k_{sys}$ and the quality factor $Q$ are taken from ready tables [14]. An example of second order MEM filter is shown in Fig. 1.

In this paper, a new approach for the coupling spring design will be presented. This approach utilizing circuit theory knowledge will use electrical equivalent models of the resonator linked with mechanical parameters as formulated previously in [15]. Both series and parallel equivalent circuits are studied. Designs and simulations presented throughout the paper were performed using MEMCAD®, Hspice®, and Matlab® software tools.

This paper starts with a complete treatment of series RLC equivalent circuit in terms of derivation of frequency response in Section 2 followed by similar approach for defining a parallel RLC equivalent circuit in Section 3. The link between electrical coupling coefficients as used in above sections and mechanical spring dimensions is provided in Section 4. Then, a complete filter design method is summarized in Section 5 along with an example for series equivalent in Section 6. Final remarks are concluded in Section 7.

2. Equivalent representation in terms of series RLC

We have chosen laterally driven MEM resonators and filters as presented by Nguyen [15] as shown in Fig. 1, for performing numerical analysis and obtaining simulation results, in this study.

The electrical equivalent model of the laterally driven filter of Fig. 1 is shown in Fig. 2. The equivalent RLC values are related with filter’s physical parameters as follows:

$$\eta = V_{p} \frac{\partial C}{\partial x}$$

$$C = \frac{\eta^2}{k_{sys}}$$

$$L = \frac{m}{\eta^2}$$

$$R = \sqrt{\frac{mk_{sys}}{Q\eta^2}}$$

(1)

where the mechanical device parameters are derived in [15]. Some parameters of Eq. (1) have been shown to be directly related to device’s physical dimensions as follows:

$$k_{sys} = 2Eh (W/L)^3$$

$$Q = \frac{d}{\mu A_e} \sqrt{\frac{mk_{sys}}{\eta^2}}$$

$$f_{res} = \frac{2Eh (W/L)^3}{3P + 0.374M} \frac{\partial C}{\partial x}$$

$$\alpha = \frac{2\eta V_{p} g}{\eta^2}$$

(2)

Detailed explanations of each of the term along with the values used in Eqs. (1) and (2) above are tabulated in Table 1. One important point to emphasize is that the mass $m$ in Eq. (1) is the combined mass of the shuttle $M_p$, truss and beam $M$, which are explained in Table 1.

The values presented solely correspond to the resonator whereas the filter parameters namely the electrical and mechanical representations of coupling spring will be presented after a design example. The frequency response of a single resonator designed mechanically for 719 kHz is shown in Fig. 3. We will now present analytical studies of series RLC.
2.1. Power transfer function

Relevant equations will be presented which are derived as part of power transfer function (henceforth termed as transfer function) evaluation and the establishment of relationships between bandwidth and the insertion loss. For the transfer function evaluation, we will consider a second order system. The equivalent circuit is given in Fig. 2.

From Fig. 2 we can single out the two individual LC tank circuits, each corresponding to an electrically driven mechanical resonator coupled via $C_{spring}$, which is the electrical equivalent of the mechanically coupled spring. Some definitions are presented in the following to simplify the transfer function calculation:

$$X := -\frac{1}{\omega C_{spring}}$$

$$B_k := \omega C_{spring}$$

Where $X$ is the complex impedance of the tank, $X_0$ is the complex impedance and $B_k$ is the complex admittance of coupling/tether spring. We can write the impedance matrix $Z$ of the form with $U_2 = -U_2/R$ and $U_2 = (U_0 - U_1)/R$ and the transfer function as $|H|_2^2 = |U_2/U_0|_2^2$. Using the definition we can now derive the transfer function of the circuit in Fig. 2 as follows:

$$U_1 = jX + X_0X_1 + jX_2X_2$$

$$U_2 = jX_0X_1 + j(X + X_0)X_2$$

The simplified magnitude of transfer function using the definitions of Eqs. (3) and (4) is then written in the following form:

$$|H(f)|_2^2 = \frac{1}{1 + [(RB_{spring}/2) + (X'/R) + (X^2 B_{spring}/2R)]^2}$$

(5)

Having established the transfer function relationship in terms of circuit components in Eq. (5) we shall relate it to the quality factor $Q_E$ of a single resonator and normalized coupling coefficient $\kappa$ of the filter. $\kappa$ is linked with $C_{spring}$ in Eq. (6) and hence, plays a central role in filter design. Some abbreviations are needed for simplifying the relations and are summarized in the following:

$$X := \frac{X_0}{R} = \frac{1}{RB_0}$$

$$\nu := \frac{\partial}{\partial \nu_0} - \frac{\partial}{\partial \nu_0}$$

(6)

$$Q E \nu = \frac{X}{R} \left( \frac{\partial}{\partial \nu_0} - \frac{\partial}{\partial \nu_0} \right)$$

(7)

$$\frac{\partial}{\partial \nu_0} = \frac{1}{\sqrt{LC}}$$

(8)

where the normalized frequency deviation $\nu$ introduced in Eq. (7) is obtained from [16]. The final transfer function is:

$$|H(f)|_2^2 = \frac{1}{1 + [(1/2\kappa) - Q_E \nu + 1/2\kappa(Q_E \nu)^2]^2}$$

(9)
2.2. Coupling coefficient limitations

The transfer function can be tuned by the coupling coefficient \( \kappa \), however it has to be bounded in order to establish appropriate operational boundaries when choosing the frequency response. Derivating Eq. (9) with respect to \( \nu \) provides the limits for \( \kappa \) which will correspond to the center and corner points in frequency. The objective is to find the range in which \( \kappa \) can be tuned to obtain selective bandwidth.

The \( \nu \) points are computed by setting:

\[
\frac{\partial (|H|^2)}{\partial \nu} = 0
\]

which yields:

\[
\nu_1,2 = \frac{\kappa}{Q_E} \pm \frac{1}{Q_E \sqrt{\kappa^2 - 1}}
\]

\[
\nu_3 = \frac{\kappa}{Q_E}
\]

All the three points corresponding to \( \nu_1,2 \) and \( \nu_3 \) (two for \( P_{\text{max}} \) and one for \( P_{\text{max}} \) in Fig. 4, respectively) are now calculated. For the transfer function to be maximally flat, requires to have \( P_{\text{min}} / P_{\text{max}} = 1 \) which is only possible when \( \kappa = 1 \). This case is called the critically coupled case and it marks the lower limit since our objective is to start from the maximally flat response and work towards bandwidth broadening. For \( \kappa > 1 \) or over-critically coupled case, the power magnitude corresponding to \( \nu_3 \) point is presented in the following:

\[
|H(\nu_3)|^2 = \frac{1}{1 + (1/2\kappa - (1/2\kappa)^2 \nu^2)}
\]

We can use Eq. (11) to evaluate the upper limit of \( \kappa \), which marks the half power point (or the 3 dB bandwidth \( \text{BW}_{3\text{dB}} \)) as \( |H(\nu_3)|^2 = 1/2 \). Whereas, \( \kappa \) can be tuned to obtain any range of insertion loss. The upper limit for \( \kappa \) could be found as:

\[
k_{1,2} = -1 \pm \sqrt{2}
\]

\[
k_{3,4} = 1 \pm \sqrt{2}
\]

With the condition \( \kappa \geq 1 \) as the lower limit, from Eq. (12) the only valid choice for the upper limit is \( 1 + \sqrt{2} \). So, coupling coefficient limits are established as:

\[
1 \leq \kappa \leq 1 + \sqrt{2}
\]  

The effects of varying \( \kappa \) onto the transfer function with respect to normalized frequency \( \nu \) are shown in Fig. 5. We can see that by changing the value of coupling coefficient, the transfer function changes in frequency domain. The maximum flat response for the \( \kappa = 1 \) is also illustrated. For the case \( \kappa < 1 \) (\( \kappa = 0.8 \) in Fig. 5), the insertion loss is considerably increasing which further strengthens the requirement of \( \kappa \) being greater than 1.

2.3. Valid cases for half power point calculations

General bandpass filter characteristics with important parameters are presented in Fig. 4. The power corresponding to the center frequency \( f_0 \) is named as the saddle point \( P_{\text{sp}} \).

In Fig. 4, \( P_{\text{max}} \) stands out as the maximum power for a single resonator, which is used in calculating the insertion loss. A relationship is required which should give us control over the bandwidth change as \( \kappa \) varies. Starting with Eq. (9) we find a closed form relationship for the bandwidth \( \text{BW}_{3\text{dB}} \) of the filter in terms of resonator quality factor \( Q_E \), normalized deviation \( \nu \) and the coupling coefficient \( \kappa \) from the transfer function as evaluated in Eq. (9). Mathematically this implies:

\[
|H(f)|^2 = \frac{1}{2} + \frac{1}{2Q_E \nu^2} \left( 1 + \left( \frac{1}{2\kappa} - Q_E \nu + \frac{1}{2\kappa} (Q_E \nu)^2 \right)^2 \right)
\]

The quadratic Eq. (14) now yields the conditions for solution of bandwidth, which leads to two cases as:

\[
\frac{\nu^2 Q_E^2}{2\kappa} - vQ_E + \frac{1}{2\kappa} = \pm 1
\]

The case \( \nu^2 Q_E^2 / 2\kappa - vQ_E + 1 / 2\kappa = +1 \) leads to two cases as:

\[
\frac{\nu^2 Q_E^2}{2\kappa} - vQ_E + \frac{1}{2\kappa} = -1
\]
Here, we can observe that the function \( f(\kappa) = \kappa^2 - 2\kappa - 1 \) becomes negative for values equal to or greater than 1 as shown graphically in the Fig. 6. The negative function results in a \( f(\kappa) \) function versus \( \kappa \) for case: +1.

\[
\Rightarrow v_1, 2 = \kappa \pm \frac{1}{Q_E} \sqrt{\kappa^2 - 2\kappa - 1} \tag{16}
\]

Here, we can observe that the function \( f(\kappa) = \kappa^2 - 2\kappa - 1 \) becomes negative for values equal to or greater than 1 as shown graphically in the Fig. 6. The negative function results in a complex normalized frequency shift from Eq. (16), which is not physically possible. Hence, first case is discarded based on the fact that we must take into account physically coupled filter design which is evaluated in the previous section in Eq. (13) and defines the minimum allowed value of \( \kappa \).

\[
\Rightarrow v_1, 2 = \kappa \pm \frac{1}{Q_E} \sqrt{\kappa^2 + 2\kappa - 1} \tag{17}
\]

The function \( f(\kappa) = \kappa^2 + 2\kappa - 1 \) is plotted in Fig. 7. The results allow the inclusion of \( \kappa \geq 1 \) and hence filter design as governed by Eq. (13) is supported by this case. We will derive bandwidth from the above-mentioned considerations and Eq. (18) in the next section.

2.4. Bandwidth (BW\(_{3\text{dB}}\)) calculation

The bandwidth is defined as \(|f_2 - f_1|\) where \( f_2 \) and \( f_1 \) are frequency points where \(|H(f)| = 1/2\). We shall now relate \( Q_E \) and \( \kappa \) to the filter bandwidth BW\(_{3\text{dB}}\).

We have already established the roots values of \( v \) in Eq. (16), which will offer valid \( \kappa \) values for the design. These roots are subtracted and their magnitude is obtained in Eq. (19), which will be used in comparison with the approximation of Eq. (20).

\[
|v_1 - v_2| = \frac{2}{Q_E} \sqrt{\kappa^2 + 2\kappa - 1} \tag{19}
\]

Where the approximation is [16]:

\[
v_1 - v_2 = \frac{\omega_1 - \omega_{\text{max}}}{\omega_{\text{res}}} - \frac{\omega_2 - \omega_{\text{max}}}{\omega_{\text{res}}} \approx \frac{\omega_{\text{max}} - \omega_1}{\omega_{\text{res}}} - 2 \frac{\omega_{\text{max}} - \omega_2}{\omega_{\text{res}}} \tag{20}
\]

Then, by combining Eqs. (19) and (20) we can obtain the following relationship for the bandwidth:

\[
\Rightarrow \frac{2}{f_{\text{res}}} |f_2 - f_1| = \frac{2}{Q_E} \sqrt{\kappa^2 + 2\kappa - 1} \tag{21}
\]

It is important to observe that the ratio \( f_{\text{res}}/Q_E \) corresponds to the bandwidth of a single resonator BW\(_E\). Hence, we can relate the bandwidth BW\(_{3\text{dB}}\) of a MEM filter to the bandwidth BW\(_E\) of a single resonator and the coupling coefficient \( \kappa \) as presented in Eq. (21). BW\(_{3\text{dB}}\) can be controlled by tuning \( \kappa \). Since, an increase in bandwidth accompanies a decrease in saddle point \( P_m \) of Fig. 4 we need to limit BW\(_{3\text{dB}}\) to a maximum allowed value, which will offer a negligible power loss at the output. The criteria for choosing the limits comes from already established boundaries of \( \kappa \) in Section 2.2 and will be used for evaluation of limiting cases of BW. Generally, there are two important constraints imposed on \( \kappa \) which may help the designer in obtaining the required passband response. They are as follows:

The smallest allowed value of \( \kappa \) should ensure \( P_m/P_{\text{max}} = 1 \), which is the critically coupled case and BW should reflect that value. If \( \kappa \) is made smaller than one, the insertion loss again increase as demonstrated in Fig. 5 for \( \kappa = 0.8 \).

The largest allowed value of \( \kappa \) should not lead to \( P_m/P_{\text{max}} < 0.5 \) because that would correspond to a signal loss greater than 3dB which is the overcoupled case. This should also mark the maximum obtainable bandwidth.
3. Equivalent representation in terms of parallel RLC

We will now present the on-going discussion implemented on a parallel RLC circuit. The force–current analogy presented in Table 2 can be used to obtain a parallel equivalent circuit representation. We summarize the parallel electrical equivalent in the following [15]:

\[
\begin{align*}
\eta & = V_A \frac{dC}{dt} \\
C & = \frac{m}{\kappa} \\
L & = \frac{Q_{sys}}{\omega_{sys}} \\
R & = \frac{Q_{sys}}{\omega_{sys}}
\end{align*}
\]

In Eq. (24) the \( L, C \) and \( R \) values correspond to parallel equivalent circuit. New notations for representing circuit elements of Fig. 8 are avoided as the series equivalent will not be discussed henceforth for bandwidth evaluation. Similar to the series RLC case, \( m \) is the total mass and \( \omega_0 \) is the center-frequency frequency of the resonator.

![Fig. 8. Electrical equivalent circuit of second order MEM filter.](Image)

3.1. Power transfer function

We will use similar approach for deriving the transfer function and the bandwidth of the equivalent circuit given in Fig. 8 as it was used in series RLC circuit.

In Fig. 8, the resonators are represented as parallel tank circuits and coupling is achieved via \( l_{spring} \). Some definitions are presented in the following to simplify the transfer function calculation.

\[
Y_A := \frac{-j\omega L_A C}{j\omega L} \\
Y_B := \frac{1}{j\omega l_{spring}}, \quad X_B := -j\omega l_{spring}
\]

where \( Y_A \) is the complex admittance of the tank and \( Y_B \) is the complex admittance of coupling/tether spring. We can write the admittance matrix \( Y \) with \( \eta = -V_2/R \) and \( I_1 = (U_1 - U_3)/R \) and the transfer function as \( |H(f)|^2 = |U_2/(U_3/2)|^2 \). Using the definitions in Eqs. (25) and (26) along with simplified complex quantities as \( Y_A = -j Y_B \) and \( Y_B = -j Y_A \), we can now derive the transfer function of the circuit in Fig. 8 as:

\[
|H(f)|^2 = \frac{1}{1 + \left[ (G/2Y) + (Y_A/G) + (Y_B^2/2GV_B) \right]^2}
\]

Having established the transfer function relationship in terms of circuit components in Eq. (27) we shall relate it to \( Q_L \) and \( \kappa \). Some abbreviations are needed for simplifying the relations and are summarized in the following:

\[
\kappa := \frac{G}{T_B} = -GR_B \\
vQ_L = \frac{Y_A}{G} = -R \sqrt{C \omega_{sys} - \frac{\omega_{sys} - \omega_{max}}{\omega_{max}}} \\
\omega_{max} = \sqrt{LC}
\]

where the parameter \( v \) is used from Eq. (7). Using these abbreviations we can write the final transfer function as presented in Eq. (30), which interestingly enough, remains unchanged as well as \( Q_L \). However, it should be kept in mind that this equivalent circuit is the dual counterpart to the series RLC circuit. Therefore necessarily the values of \( R, L \) and \( C \) differ.

\[
|H(f)|^2 = \frac{1}{1 + \left[ \frac{1}{Q_L} - Q_L v + \frac{1}{\sqrt{2}} (Q_L v)^2 \right]^2}
\]

The remaining calculations will show no deviation from the previous case and will be summarized only.

3.2. Coupling coefficient limitations

The objective, once again, is to bind \( \kappa \) within limits to have a starting point estimate while beginning the design. Deriving
Eq. (30) with respect to \( \nu \), yields the roots which correspond to the maximum and 3 dB-point.

\[
\nu_1, 2 = \left( \frac{\kappa}{Q_E} \right) \pm \frac{1}{Q_E} \sqrt{\kappa^2 - 2\kappa - 1}
\]

For the transfer function the points for \( P_{\text{max}} \) and \( P_m \) have been calculated. The limits for \( \kappa \) are presented in Eq. (13).

The studied effects of varying onto the parallel transfer function and the graphical justification of upper and lower limits of \( \kappa \) is analogous to as shown in Fig. 5. The similarity which makes sense as both the equivalent representations are for the same mechanical structure; it is just the matter of choice imposed by available simulators and resonator structures.

3.3. Valid cases for half power point calculations

In order to evaluate frequency points which correspond to \( P_{\text{max}} \) and \( P_m \), we start with the transfer function Eq. (30) for the parallel RLC circuit and represent it in the terms of \( Q, \nu \) and \( \kappa \). Solving Eq. (31) yields the solution for the two cases as presented in the following:

\[
|H(f)|^2 = 1 \Rightarrow 1 + \left[ \frac{1}{2\kappa} - Q_E^2 + \frac{1}{2\kappa}Q_E^2\nu^2 \right]^2 = 2 \quad (31)
\]

3.3.1. First case

\[
\Rightarrow \nu_{1,2} = \frac{\kappa}{Q_E} \pm \frac{1}{Q_E} \sqrt{\kappa^2 - 2\kappa - 1} \quad (32)
\]

3.3.2. Second case

\[
\Rightarrow \nu_{1,2} = \frac{\kappa}{Q_E} \pm \frac{1}{Q_E} \sqrt{\kappa^2 + 2\kappa - 1} \quad (33)
\]

The cases are identical to the series counterpart. Taking into account the behavior of \( \kappa \) in Eqs. (32) and (33) as well as the allowed boundary points for \( \kappa \) in Eq. (13) we can validate the second case to be used in 3 dB bandwidth calculations.

3.4. Bandwidth (BW 3dB) calculation

For the parallel RLC circuit we will use the approximation of Eq. (20). Using the results of Eq. (33) the final equation for the bandwidth is

\[
|f_1 - f_2| = \frac{f_{\text{res}}}{Q_E} \sqrt{\kappa^2 + 2\kappa - 1} \quad (34)
\]

Depending on the operating environment (vacuum or atmosphere), the designer may choose to work with series RLC circuit or parallel RLC circuit. For low source/load impedance designs, series RLC is preferred over its dual counterpart.

4. Electro-mechanical model of coupling spring

In this section, we will link mechanical and electrical representations of coupling phenomena as evaluated by

\[
k_{\text{spring}} = \frac{\kappa}{Q_E} \quad (35)
\]

\[
k_{\text{spring}} = 2Eh_{\text{spring}} \frac{w_{\text{spring}}}{t_{\text{spring}}} \quad (36)
\]

whereas \( k_{\text{spring}} \) is the stiffness constant, \( V_p \) is the applied dc voltage, \( \theta C/\kappa \) is the differential change in the coupling capacitance per displacement already introduced in Table 1.
As seen in Eq. (35) \( k_{\text{spring}} \) is inversely proportional to \( C_{\text{spring}} \). The stiffness constant can therefore be taken as a coupling factor and can be used in tuning \( C_{\text{spring}} \) or vice versa to obtain desired transfer function characteristics.

For the coupling spring design the circuit analysis yields a design criterion for \( C_{\text{spring}} \). Then \( k_{\text{spring}} \) can be computed by using Eq. (35). The found value for \( k_{\text{spring}} \) is substituted in Eq. (36) and the spring length \( l_{\text{spring}} \) can be calculated. The calculation of \( l_{\text{spring}} \) is based on pre-defined values for the structural thickness \( h_{\text{spring}} \) and the spring width \( w_{\text{spring}} \) which are technology dependent. In this study \( h_{\text{spring}} \) and \( w_{\text{spring}} \) are set to 2 and 3 \( \mu \)m, respectively.

### 5. Design summary

As previously mentioned, the filter design utilizes the mechanical properties as well as the electrical equivalent. Therefore, the proposed design starts in the mechanical domain and after being fine-tuned in electrical domain, is again converted into its mechanical equivalent with calculated physical parameters. The constituent resonators are designed mechanically and the coupling is studied electrically. Let us summarize the filter design methodology with the flow chart in Fig. 10:

1. Starting from given specifications of center frequency and bandwidth requirements. Design a mechanical resonator, which satisfies the center frequency. Symmetric resonators will be coupled so that the designed resonators should have maximally flat passband or Butterworth like response. \( B_{\text{AWB}} \) requirement is addressed in the electrical domain.
2. Establish the working environment (e.g. atmospheric pressure/vacuum, applied dc/ac voltage levels, gap to substrate spacing, finger gaps). These values will be used in converting the designed resonator into its electrical equivalent circuit.
3. Tune the coupling spring to achieve the required filter bandwidth \( B_{\text{3dB}} \) while staying within the bounds of Eq. (13). Note that this boundary is only a suggestion, not a necessity. This tuning will provide a value for \( C_{\text{spring}} \) using Eq. (6). It should also be noted that as \( \kappa \) is chosen to be frequency independent in the passband, \( \omega_{\text{m}} \), the center frequency of the filter would be used in place of \( \omega \). With the value of \( C_{\text{spring}} \) known, we can calculate the mechanical spring coupling \( k_{\text{spring}} \). This value of \( k_{\text{spring}} \) is matched with the mechanical coupling coefficient as given by Eq. (36) or whichever coupling coefficient equation is valid for a given device geometry. Hence, knowing the value of \( \kappa \) which satisfies all the specifications, physical dimensions of the coupling beam can be calculated.
4. By adjusting the process dependent parameters (i.e. in this study, width and height of coupling beam \( w_{\text{spring}}, h_{\text{spring}} \) respectively) mechanical realization of the coupling spring is possible and ultimately the design returns back to the mechanical domain.

### 6. Design example

Simulations with a commercial tool are performed to verify the design of mechanically coupled bandpass structures using the above presented method. The simulation results for two cases (\( \kappa = 1 \) and \( \kappa = 2.414 \)) at about \( f_{\text{m}} = 719 \) kHz are presented in Fig. 11, with Table 3 summarizing the simulation data.

<table>
<thead>
<tr>
<th>( f_{\text{m}} ) (kHz)</th>
<th>( \kappa )</th>
<th>( B_{\text{3dB}} ) (Hz, %)</th>
<th>( k_{\text{spring}} ) (N/m)</th>
<th>( C_{\text{spring}} ) (fF)</th>
<th>( l_{\text{spring}} ) (( \mu )m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>719.115</td>
<td>1</td>
<td>75/0.0104</td>
<td>0.123</td>
<td>2.21</td>
<td>508</td>
</tr>
<tr>
<td>719.095</td>
<td>2.414</td>
<td>165/0.00229</td>
<td>0.296</td>
<td>0.92</td>
<td>379</td>
</tr>
</tbody>
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C 

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der MEM bandpass filters using hybrid mechanical/electrical 
tuned to obtain the desired response.

As the coupling spring design does not include the effects of series resistance, a value of R representing operating conditions under vacuum is used. Due to changing Q values, κ may not remain bounded by Eq. (13). The bandwidth can be increased by sacrificing a high Q 

The effects of insertion loss IL and series R are discussed in [18] where a filter operating at 840 kHz with a bandwidth ranging from 0.02 to 0.08% is demonstrated. It is possible that while including the effects of R, the bandwidth may increase owing to the reduced Q. The coupling in that case can be fine tuned to obtain the desired response.

7. Conclusion

We have presented a design methodology for second or-
der MEM bandpass filters using hybrid mechanical/electrical 
domain formulations. We have addressed the power transfer 
function of the mechanical filters which can be represented 
both as series and parallel LC tanks circuits using network 
theory. Consequently, the transfer functions are ultimately 
used for finding closed form relationships for bandwidth in 
terms of normalized coupling coefficient κ or coupling ca-
pacitance Cspring. The coupling coefficient is shown to be 
bound within a range, which provides two different pass-
band responses in the filters namely maximally flat and equi-
ripple. Finally, the κ as well as Cspring from electrical design 
is linked with mechanical parameters, for instance coupling 
spring dimensions. The filter design is cooped up in a sum-
mary and a complete design example is presented to illustrate 
the methodology developed in this study.

It is necessary to mention at this conjuncture that the filter design considered in the sections above can be looked upon as exhibiting Butterworth and Chebyshev like filter behaviors due to similar transfer function formulae and behavior [19]. This is only a qualitative observation and part of future work may include some quantitative approaches to prove that the filter indeed exhibiting a response identical to Butterworth and/or Chebyshev filter transfer characteristics. Furthermore, much work is already underway towards increasing the filter order for improved selectivity [12]. Since, wireless systems could demand higher order (more than fourth) structures than already presented in this study, theory, presented in this pa-

der paper shall be extended to incorporate the effects of three or more mechanical resonators coupled to form bandpass filter characteristics.

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