Coupling Spring Design and Insertion Loss Minimization of Second Order MicroElectroMechanical Filters

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ABSTRACT

For a microelectromechanical (MEM) filter a new approach for the coupling spring design is presented. A designed MEM bandpass filter consists hereby of two laterally driven resonance structures which are coupled by a mechanical spring. For this structure an electrical equivalent circuit is presented and analyzed analytically. The transfer function and the correlation between the bandwidth and the coupling spring for this filter are derived. A complete design procedure for micromechanical bandpass filters is presented with simulation results. With changing design parameters the filter response has been demonstrated to be maximally flat as in the case of Butterworth-Filters, or to have a response allowing a wider bandwidth than in the former case. Both of these cases are studied theoretically and simulated for the same frequency range. In respect of the insertion loss value, a modified source and load resistance is applied which keeps it under 1 dB.

Keywords: MEMS, resonator, bandpass filter, coupling

1 INTRODUCTION

Laterally driven MicroElectroMechanical (MEM) resonator based filters are considered as potential replacements of prevalent tank circuits due to their very high quality factor and the chip integration compatibility. Presently, these MEM filters are incompatible for the industry as compared to their off-chip counterparts which have obtained industrial standard due to proper insertion loss and bandwidth. However, MEM filters have been demonstrated to have a very high quality factor (as high as 80,000 under vacuum [1]) which allows greater selectivity. Micromachined bandpass filters have been demonstrated both in high frequency (e.g. 14.5 MHz [2]) and intermediate frequency regions (e.g. 455 kHz [3]) with achieved bandwidths in the order of 0.2% and 0.09% respectively. The insertion losses were as small as 1 dB. For these designs both the coupling spring and the insertion loss should be taken into account. The design of MEM filters follow a cook book approach in which the coupling spring coefficient $k_{spring}$ and the quality factor $Q$ are taken from ready tables [4].

A bandpass structure based on two coupled resonators is investigated. Hereby both resonators are identical. This requirement complies with preferable practical implementations of planar IC processes [3]. In this paper a new approach for the coupling spring design will be presented. The circuit theory approach utilizes an electrical equivalent model of the resonators linked with mechanical parameters as formulated previously in [5].

2 COUPLING SPRING DESIGN

2.1 The Electrical Equivalent Circuit

Mechanical bandpass filters are obtained by coupling e.g. two resonators through a spring. Figure 1 shows a 3-dimensional view of the designed MEM bandpass filter. An AC signal is applied at the drive electrode and the resulting response is detected at the sense electrode. Two identical resonators are coupled with one tether spring $C_{spring}$.

![Figure 1: The 3-dimensional view of a mechanically coupled bandpass filter](image)

An electrical equivalent representation has been evaluated in [5] to model the mechanical components. Figure 2 depicts the electrical equivalent circuit. Each series $L/C$ tank circuit corresponds to an identical resonator. The coupling spring corresponds to the capacity $C_{spring}$. 
2.2 Mechanical and Electrical Equivalent Representation of the Coupling Spring

The relation between the coupling spring stiffness $k_{spring}$ in the mechanical design and the capacitance $C_{spring}$ in the equivalent electrical circuit is provided by the following Equations (1) and (2) [6].

$$k_{spring} = \frac{h_{spring}^2}{C_{spring}} = \frac{(U_{bias} \cdot \frac{\partial C}{\partial x})^2}{C_{spring}}$$ (1)

$$k_{spring} = 2 \cdot E \cdot h_{spring} \cdot \left(\frac{w_{spring}}{l_{spring}}\right)^3$$ (2)

whereas $k_{spring}$ is the stiffness constant, $U_{bias}$ is the applied DC voltage, $\frac{\partial C}{\partial x}$ is the differential change in the coupling capacitance per displacement, $h_{spring}$, $w_{spring}$ and $l_{spring}$ are the height, the width and the length of the mechanical spring, respectively and $C_{spring}$ is the value for the coupling capacitance in the equivalent circuit. $E$ is a constant known as the Young’s modulus and taken as 150 GPa in this study.

As seen in Equation (1) $k_{spring}$ is inversely proportional to $C_{spring}$. The stiffness constant can therefore be taken as a coupling factor as shown in the next section.

For the coupling spring design the circuit analysis yields a design criteria for $C_{spring}$. Then $k_{spring}$ can be computed by using Equation (1). The found value for $k_{spring}$ is substituted in Equation (2) and the spring length $l_{spring}$ can be calculated. The calculation of $l_{spring}$ is based on pre-defined values for the structural thickness $h_{spring}$ and the spring width $w_{spring}$ which are technology dependent. In this study $h_{spring}$ and $w_{spring}$ are set to 2 $\mu$m and 3 $\mu$m respectively.

2.3 Filter Bandwidth

In order to find a relation between the coupling spring capacitance $C_{spring}$ and the 3 dB bandwidth $BW_{3dB}$ of the filter only the $L_xC_x$ tank circuits with $C_{spring}$ are being considered. The value of $R_x$ is firstly neglected to keep the equations simple by assuming $R_x \approx 0$. The equivalent circuit then simplifies as presented in Figure 3.

$$\omega_{res} = \frac{1}{\sqrt{L_xC_x}}$$ (3)

$$X_x = \omega L_x - \frac{1}{\omega C_x}$$ (4)

$$B_{spring} = \omega C_{spring}$$ (5)

$$Q_e = \frac{1}{\omega_{res}RC_x} = \frac{\omega_{res}L_x}{R} = \frac{1}{R} \sqrt{\frac{L_x}{C_x}}$$ (6)

$$BW_e = \frac{f_{res}}{Q_e}$$ (7)

$$|H(f)|^2 = \frac{1}{1 + \left[\frac{R B_{spring}}{2} + X_x + \frac{X_x^2 B_{spring}}{2R}\right]^2}$$ (8)

whereas $\omega_{res}$ is the angular resonance frequency for one series resonance circuit, $X_x$ is the reactance of the resonance circuit, $B_{spring}$ is the capacitive susceptance of $C_{spring}$ and $Q_e$ is the loaded quality factor of the resonance circuit due to $R$. Furthermore, $BW_e$ is the 3 dB bandwidth of one resonance circuit. Here, the transfer function $H(f) = U_2/U_1$ is conveniently presented as the squared magnitude $|H(f)|^2$ and corresponds to the power transfer function.

To derive the 3 dB bandwidth $BW_{3dB}$ of the bandpass filter the abbreviations $\kappa$ as the coupling factor and the normalized frequency deviation $\nu$ [7] are defined as followed.

$$\kappa := \frac{1}{R B_{spring}} = \frac{1}{R \omega C_{spring}}$$ (9)

$$\nu := \frac{\omega_{res}}{\omega} - \frac{\omega}{\omega_{res}}$$ (10)

Hence the power transfer function can be written in terms of $\kappa$ and $\nu$ as
\[ |H(f)|^2 = \frac{1}{1 + \left( \frac{1}{2 \kappa} - Q_e \nu + \frac{1}{2 \kappa} (Q_e \nu)^2 \right)^2} \]  
(11)

By the inspection of Equation (1) and Equation (9) the coupling factor \( \kappa \) for the electrical circuit and the coupling coefficient \( k_{spring} \) for the mechanical structure are directly corresponding. For \( \kappa < 1 \) it is an undercritically coupled filter, for \( \kappa = 1 \) or \( \kappa > 1 \) the filter is critically or overcritically coupled respectively. Considering low insertion losses only the two latter mentioned cases are of interest.

The coupling factor \( \kappa \) can be considered frequency independent within the small filter bandwidth. Also by using the Taylor expansion to simplify the normalized frequency deviation \( \nu \), the relationship between the 3 dB bandwidth \( BW_{3dB} \) of the bandpass filter and the coupling factor \( \kappa \) is

\[
BW_{3dB} = \frac{f_{res}}{Q_e} \sqrt{\kappa^2 + 2 \kappa - 1}
\]  
(12)

The 3 dB bandwidth \( BW_{3dB} \) is defined at those frequency points when the voltage amplitude is 3 dB lower than the maximum voltage \( U_{\text{max}} \) as depicted in Figure 4.

\[ f_m < \kappa < 1 + \sqrt{2} \]  
(13)

\[ \sqrt{2} \cdot BW_e < BW_{3dB} < 3.108 \cdot BW_e \]  
(14)

### 3 SIMULATION

With a commercial simulation tool several simulations are performed to verify the design of mechanically coupled bandpass structures using the above method. The simulation results for two cases (\( \kappa = 1 \) and \( \kappa = 3.108 \)) at about \( f = 840 \text{ kHz} \) are presented in Figure 5, and Table 1 shows the summarized simulation data.

![Figure 5: Simulation result for the critically and overcritically coupled filter at \( f = 840 \text{ kHz} \)](image)

**Table 1: Summary of the simulation data**

<table>
<thead>
<tr>
<th>( f_m ) [kHz]</th>
<th>( \kappa )</th>
<th>( BW_{3dB} ) [Hz/%]</th>
<th>( C_{spring} ) [F]</th>
<th>( l_{spring} ) [( \mu \text{m} )]</th>
</tr>
</thead>
<tbody>
<tr>
<td>840.30</td>
<td>1</td>
<td>243.5 / 0.0290</td>
<td>1.62</td>
<td>364</td>
</tr>
<tr>
<td>840.55</td>
<td>3.108</td>
<td>728.0 / 0.0866</td>
<td>0.673</td>
<td>271</td>
</tr>
</tbody>
</table>

With \( R = 12.0 \text{ M}\Omega \), \( R_x = 116.54 \text{ M}\Omega \), \( L_x = 78875 \text{ H} \), and \( C_x = 4.5485 \times 10^{-19} \text{ F} \)

For these two presented cases reasonable coupling spring lengths \( l_{spring} \) are obtained by using the method established in Section 2.3. The percentage bandwidth is defined as \( BW_{3dB}/f_m \cdot 100\% \). Further necessary values are the bias voltage \( U_{\text{bias}} = 35 \text{ V} \) and \( \partial C/\partial x = 2.59 \times 10^{-9} \text{ F/\mu m} \).

The insertion loss (IL) is \(-26 \text{ dB} \) which corresponds to a power loss by the factor of 400. It is of notable significance and is treated in the forth coming section.

### 4 INSERTION LOSS

The insertion loss is very high due to the high value of the ratio \( R_x/R \). Normally the MEM structures are
operated in vacuum which means that the value of \( R_x \) decreases drastically. Typical values are then \( R_x \approx 500 \, \text{k}\Omega \) [5]. Reasonable low insertion losses of less than 1 dB can be obtained when \( R \geq 20 \cdot R_x \). Hence it must be guaranteed that the source and load resistance values are higher than \( R_x \) of the filter. For instance, the required high source and load resistances can be realized by a current source and an electrometer amplifier respectively. Figure 6 shows the reduced insertion loss for \( R = 20 \, \text{M}\Omega \) when the structure is operated under vacuum.

![Figure 6: Simulation result for the overcritically coupled filter at \( f = 840 \, \text{kHz} \) with \( R = 20 \, \text{M}\Omega \)](image)

It is seen that the insertion loss is now less than 0.5 dB. Since the source and load resistances are changed also the coupling spring value \( C_{\text{spring}} \) has to change in order to keep the filter coupling constant. The new value for \( C_{\text{spring}} \) is 3.9 fF whereas the values for \( L_x \) and \( C_x \) are the same. Summarized are the data in Table 2.

Table 2: Summary of the data for low insertion loss

<table>
<thead>
<tr>
<th>( f_{\text{m}} ) [kHz]</th>
<th>( \kappa )</th>
<th>( BW_{\text{dB}} ) [Hz/%]</th>
<th>( C_{\text{spring}} ) [F]</th>
<th>IL [dB]</th>
</tr>
</thead>
<tbody>
<tr>
<td>840.32</td>
<td>3.108</td>
<td>84.0 / 0.010</td>
<td>3.90</td>
<td>&lt; 0.5 dB</td>
</tr>
</tbody>
</table>

with \( R = 20.0 \, \text{M}\Omega \)  
\( R_x = 500 \, \text{k}\Omega \)  
\( L_x = 78875 \, \text{H} \)  
\( C_x = 4.5485 \times 10^{-19} \, \text{F} \)

Due to the lower value of \( R + R_x \) in respect to the previous case the bandwidth is smaller. The center frequency is slightly lower.

5 CONCLUSION

A new method of mechanical coupling spring design through its electrical equivalent circuit model is established and linked with important filter parameters, e.g. the bandwidth and insertion loss. The advantage of this technique can be fully employed for the cases whenever flexible bandwidth control is part of design criteria. The design is carried out in the electrical domain which assists the fine-tuning of \( C_{\text{spring}} \) to meet the bandwidth requirement after which the circuit is optimized for reduced insertion loss. An example filter operating at 840 kHz is designed and simulation results are presented which show agreement with established theory. Future considerations include treatment of shape factor in bandpass filters and more comprehensive coverage of coupling factor \( \kappa \) and its boundary conditions.

REFERENCES