CS 290T Computational Algebra
Winter Term 2016

Homework Assignment 01:
Consider the set of linear equations with the Hilbert matrix of dimension 3

$$\begin{bmatrix} 1 & \frac{1}{2} & \frac{1}{3} \\ \frac{1}{2} & \frac{1}{3} & \frac{1}{4} \\ \frac{1}{3} & \frac{1}{4} & \frac{1}{5} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} \frac{1}{4} \\ \frac{1}{5} \\ \frac{1}{6} \end{bmatrix}$$

whose entries can be scaled up by $\mathrm{lcm}(2, 3, 4, 5, 6) = 60$ to obtain the integer system of equations

$$\begin{bmatrix} 60 & 30 & 20 \\ 30 & 20 & 15 \\ 20 & 15 & 12 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 15 \\ 12 \\ 10 \end{bmatrix}$$

The determinant and the solution can be found using Mathematica as

$$d = 100 \quad \text{and} \quad \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} \frac{1}{20} \\ -\frac{3}{5} \\ \frac{3}{2} \end{bmatrix}$$

- Solve this integer system of equations using the multiple-modulus congruence technique with the moduli set $(n_1, n_2, n_3, n_4) = (7, 9, 11, 13)$. Add one more modulus if necessary. Use the MRC algorithm at the last step.
- Solve this integer system of equations using Dixon's algorithm for $p = 13$ and as large k as needed. Determine the minimum k value which finds the solution.

Due 5pm Wednesday January 20
Either, email an electronic copy to me (koc@cs.ucsb.edu) or bring a paper copy to the class. Electronic copy of your homework can be in Text or PDF. You could also scan/pdf your handwritten work; however, do not send (low-resolution or small) phone-camera images under any circumstances! Put your name inside the file. Also make the attached file name as your last name, followed by homework number, for example: green-hw01.pdf