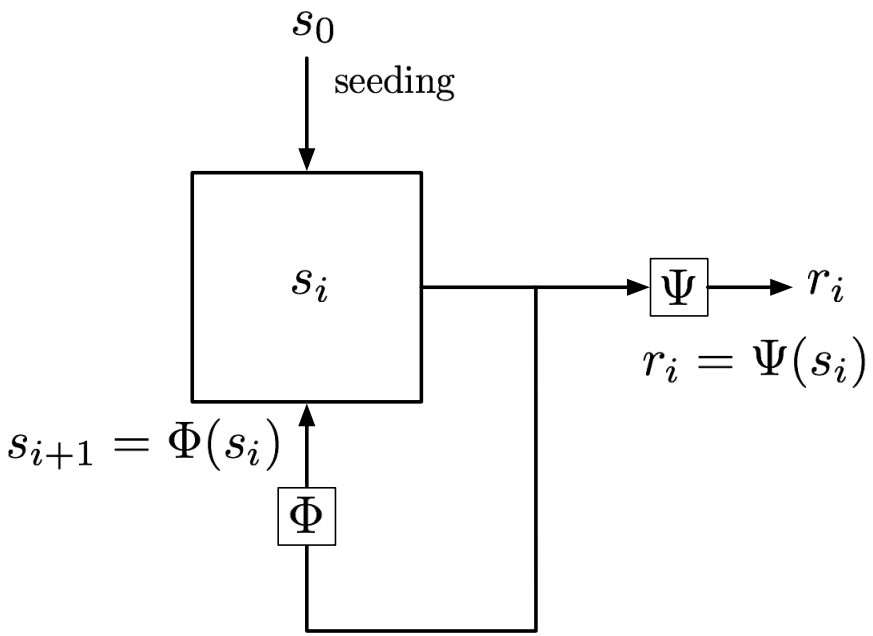
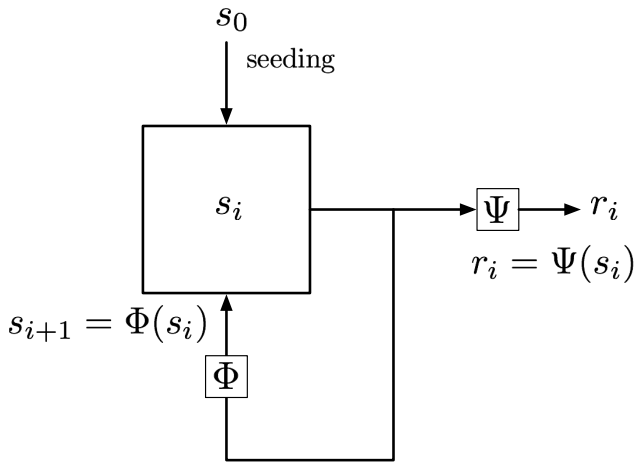
# Deterministic Random Number Generators



# DRNGs

- A pure DRNG starts with a seed ($s_0$) value and using a **seeding algorithm** computes the first internal state $s_1$ from $s_0$
- Once $s_0$ is made available the subsequent internal states $s_{i+1}$ are being produced using the **state transition function** $\Phi(s_i)$
- The output is the random number $r_i$ which is computed using the **output functions** $\Psi(s_i)$ from the internal state $s_i$

$$
\begin{aligned}
s_1 &= \text{seeding}(s_0) \\
r_i &= \Psi(s_i) \\
s_{i+1} &= \Phi(s_i)
\end{aligned}
$$



# DRNG Examples

- Linear Feedback Shift Registers (LFSRs)
- Cellular Automata (CA)
- Linear Congruential Generators (LCGs)
- Block cipher and Hash function based methods
- Number-theoretical methods: RSA, Rabin, Blum-Blum-Shub
- Elliptic curve methods: LCG, Power Generator, Naor-Reingold

# DRNG Advantages

DRNGs as random number generators have many advantages:

- low cost
- no dedicated hardware is required
- implementations can be done in software
- identical seed values imply identical random numbers which is a necessary condition for using them as stream ciphers

# DRNG Disadvantages

However, there are disadvantages:

- For pure DRNGs, the output is completely determined by the seed
- Output sequences of pure DRNGs cannot be truly independent
- They may behave as output sequence of an ideal RNG at most with respect to certain aspects
- The internal state has to be protected even if the device is not active

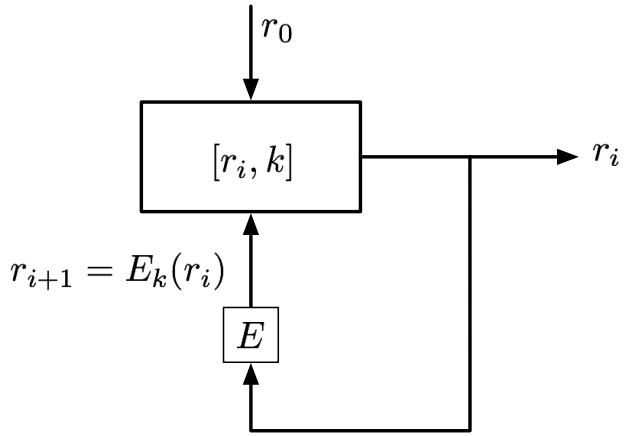
# Security Requirements of DRNGs

- LFSRs: usually meet Requirement R1, but not R2
- CA: meet R1 and R2 on certain conditions
- LCGs: usually meet Requirement R1, may meet R2
- Block cipher methods: meet R1 and R2 on certain conditions
- Hash function methods: meet R1 and R2 on certain conditions
- Number-theoretical methods: meet R1 and R2 on certain conditions
- Elliptic curve methods: meet R1 and R2 on certain conditions

- Simple structures, such as LFSRs, are useful for fast and efficient implementations but they may have serious security shortcomings

# Block Cipher DRNGs

- The seed $r_0$
- The initial state $[r_0, k]$
- The key $k$ is to be kept secret
- Internal state is the pair $s_i = [r_i, k]$
- The output $r_i$
- Next state $s_{i+1} = [E_k(r_i), k]$



# Block Cipher DRNGs

- Block cipher based DRNGs meet R1: ciphertext from a strong cipher should not have any statistical weakness
- Block cipher based DRNGs also meet R2: only if the encryption and decryption functions are secure against chosen-plaintext attacks
- This "security proof" is typical for DRNGs: tracing back to the recognized properties of well-known cryptographic primitives

# Block Cipher DRNGs

- If the key $k$ is unknown, the computation of the next state $s_{i+1} = [r_{i+1}, k]$ from the previous state $s_i = [r_i, k]$ is equivalent to breaking of the block cipher since $r_{i+1} = E_k(r_i)$
- Similarly, if the key $k$ is unknown, the computation of the previous state $s_i = [r_i, k]$ from the next state $s_{i+1} = [r_{i+1}, k]$ is equivalent to breaking of the block cipher since $r_i = D_k(r_{i+1})$

# Block Cipher DRNGs

- Therefore, unpredictability, or Requirement R2, depends on the cryptographic strength of the block cipher
- If the block cipher is secure, then Requirement R2 is satisfied
- For AES and Triple-DES encryption functions, we may assume that this type of DRNG meets R2
- In the 80s, the same conclusion was justified for Single-DES, but this conclusion is no longer valid!

# ANSI X9 Standards

- A well-known block cipher based DRNG called X9.17 is standardized for the financial industry
- American National Standards Institute (ANSI) is broken down into committees, one being ANSI X9
- The committee ANSI X9 develops standards for the financial industry, more specifically for personal identification number (PIN) management, check processing, electronic transfer of funds, etc
- Within the committee of X9, there are subcommittees
- The actual documents are named as X9.9 and X9.1, etc

# ANSI X9.17

- ANSI X9.17 defines a format for messages to establish new keys and replace old ones called CSM (cryptographic service messages)
- ANSI X9.17 also defines two-key triple-DES encryption as a method by which keys can be distributed
- ANSI X9.17 is gradually being supplemented by public-key techniques such as Diffie-Hellman encryption
- Triple DES with two keys is defined as the function

$$E_k(M) \equiv E_{k_1}(D_{k_2}(E_{k_1}(M))) \quad \text{such that} \quad k = k_1 \,||\, k_2$$

- The symbol $||$ stands for the concatenation operator

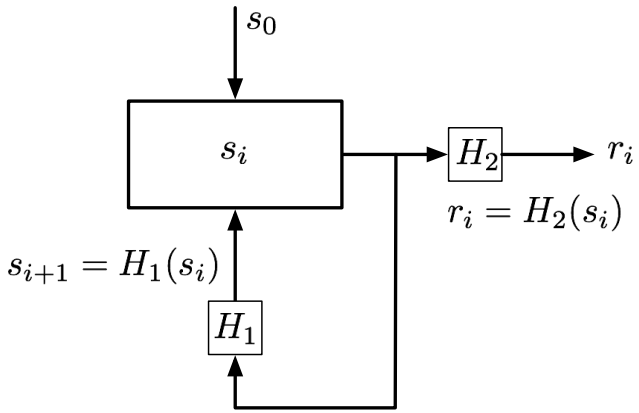
# ANSI X9.17 DRNG

- **Input:** A secret 64-bit seed $s$, 112-bit key $k$, and an integer $m$
- **Step 1:** Compute $I = E_k(D)$
  $D$ is the 64-bit representation of the date and time[1]
  $D$ is used as the initializing variable (IV)
- **Step 2:** For $i$ from 1 to $m$
  - $r_i \leftarrow E_k(I \oplus s)$
  - $s \leftarrow E_k(I \oplus r_i)$
- **Output:** Return $r_1, r_2, \ldots, r_m$

[1] The Unix Time is a 32-bit number

# Hash Function DRNGs

- The seed $s_0$
- $s_i$ is a vector of at least 256 bits
- $H_1$ and $H_2$ are two different hash functions
- Next state $s_{i+1} = H_1(s_i)$
- The output $r_i = H_2(s_i)$
- Fulfills R1 and R2



# Hash Function DRNGs

- Since $s_{i+1} = H_1(s_i)$, the computation of the next state $s_{i+1}$ from the previous state $s_i$ is trivial
- However, the computation of the previous state $s_i$ from the next state $s_{i+1}$ requires the inversion of the one-way function $H_1$
- Similarly, the computation of the output $r_i$ from the previous state $s_i$ is also trivial
- However, the computation of the previous state $s_i$ from the output $r_i$ requires the inversion of the one-way function $H_2$

# Security Requirement R4′

- Therefore, the discovery of the current state $s_i$ is detrimental to the security of the hash function DRNG
- Similarly, the discovery of the current state $s_i = [r_i, k]$ would be detrimental to the security of the block cipher DRNG

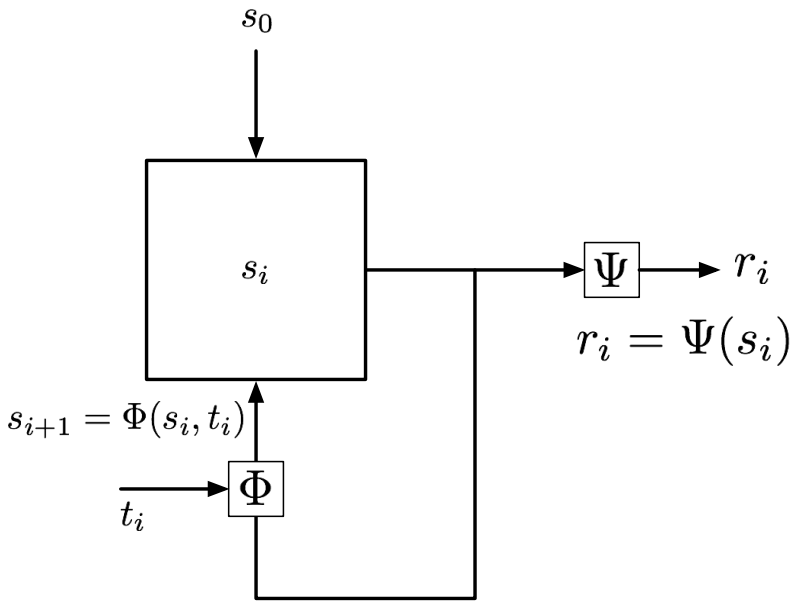
- For specific applications, Requirement R4′, named as **enhanced forward security**, is desirable
- *It should not be practically feasible to compute future random numbers from the internal state or to guess them with non-negligibly larger probability than without the knowledge of the internal state*

# Security Requirement R4′

- **Attack Scenario:** An attacker is able to read or to manipulate the internal state of a DRNG without being noticed by the user/owner of the DRNG who uses the subsequent random numbers
- Pure DRNGs cannot fulfill Requirement R4′
- A hybrid DRNG may fulfill R4′ for the random numbers that are generated after the first update of its internal state with random data after the internal state has been compromised

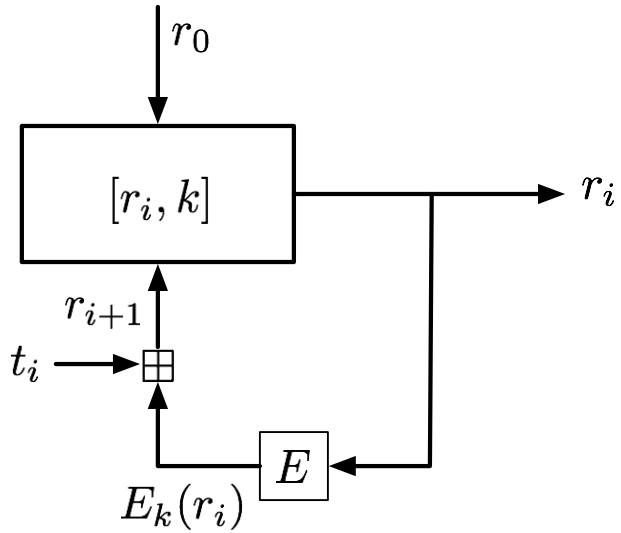
# Hybrid DRNGs

- $s_0$: Seed
- $r_i$: Random number
- $\Phi$: State transition function
- $\Psi$: Output function
- $t_i$: **Additional input**
- $t_i$: Non-deterministic data, for example, time, system data, random bits from a TRNG



# Example: Block Cipher Hybrid DRNGs

- Strong block cipher, such as AES or Triple-DES
- Key $k$ is kept secret
- $t_i$: **Additional input**
- $r_{i+1} = t_i \oplus E_k(r_i)$
- The algorithmic part guarantees R1 and R2
- Additional input with large entropy may ensure R3 and R4



# Hybrid DRNGs

- Additional input may be provided
  - After each step
  - Occasionally
  - Upon external request of an application
- Additional input may have
  - Large entropy per bit, using a strong physical RNG
  - Low entropy, time etc

# DRNGs vs TRNGs

- A pure DRNG fulfills Requirement R2 and possibly R3 if the state function and output function are sufficiently complex
- A pure DRNG provides practical (computational) security
- Its security assessment may change in the course of time

- A TRNG fulfills R2 if the entropy of the internal random numbers is sufficiently large
- This implies theoretical (time-invariant) security
- Secure forever :)

# Usefulness of Hybrid RNGs

- In practice: coming up with high quality and well controlled noise sources is challenging
- Several proposals have failed over the years, due to entropy properties or due to the use of simple structures for algorithmic post-processing
- We are stuck between a rock and a hard place:
  - DRNG: Security properties may change over time
  - TRNG: Low-cost, high-quality, and high-bandwidth entropy sources that can be integrated on to our chips are scarce
- Hybrid DRNGs: low-cost and low-bandwidth entropy sources, coupled with cryptographically strong and high-bandwidth functions for DRNGs, whose design parameters can be selected and controlled