CS 130A Data Struc & Alg 1

1. Revise all four algorithms given in the class notes for finding the maximum subsequence sum so that each one of them finds the indices (i, j) of the subsequence as well.

2. Order the following functions by their growth rate (smallest to largest)

$$n^3 \quad 2^{\sqrt{n}} \quad 10^n \quad \frac{n}{\log n} \quad 2^n \quad n\log^2 n \quad 2^{\log^2 n} \quad \frac{n^2}{\log^2 n} \quad n^{1/3}\log^3 n \quad 3^{n!}$$

3. An algorithm takes 1 second for input size 10. Estimate the number of seconds the algorithm would require on the same computer when the input size doubles (20), triples (30), and quadruples (40) if its running time is **each** of the following:
   $O(\log n)$, $O(\log^2 n)$, $O(n)$, $O(n\log n)$, $O(n^2)$, $O(n^2\log^2 n)$, $O(n^3)$, $O(2^n)$, $O(n^n)$

4. Suppose a parallel computer system has 1,000 processors which are programmed to solve a particular problem in parallel. Derive a reasonable estimate for the largest values of n this computer system can solve in 1 day, 1 month, 1 year, and 10 years, respectively, assuming the algorithms for solving this problem of size n requires $O(n)$, $O(n^2)$, $O(n^3)$ and $O(2^n)$ steps and one step takes 1 milliseconds.

5. Calculate the big-O estimates for the following functions:
   $(n^2+n)(n^2+1000)$
   $(n\log^2 n+n^2)(n^3+n^2)$
   $(2^n+n^3)(n^3+2^n)$
   $(n^3+n^2\log n)(1+\log n)+(n+\log^2 n)(n^3+1)$
   $(2^{n!}+2^{2^n})(n!+n^{n^2})$
   $(n^3\log^2 n+n^2\log^3 n)(1+\log n+\log^2 n+\log^3 n)(1+\log n^3)$
   $(n+n^2+n^3)(2^{\log n}+2^{\log^2 n}+2^{\log^3 n})$
   $(1+n+n^2)(n^3+n^4+n^5)$
   $\sum_{k=1}^{n} k^3$
   $\sum_{k=1}^{4} \log^k n$
   $\prod_{k=1}^{8}(1+\log n)^k$

6. Consider the binary exponentiation algorithm: x and e are the inputs such that x is a real number; e is a positive, nonzero n-bit integer; y is the output such that $y = x^e$.

   ```
   y = 1
   for i = n − 1 down to 0
       y = y · y
       if e_i = 1 then y = y · x
   return y
   ```

   Give the best-case, average-case, and worst-case analysis of the algorithm in terms of the number of multiplications. Note: count squarings as multiplications.

Deliver the assignment via Gradescope; link is to be provided. Late submissions are not accepted.