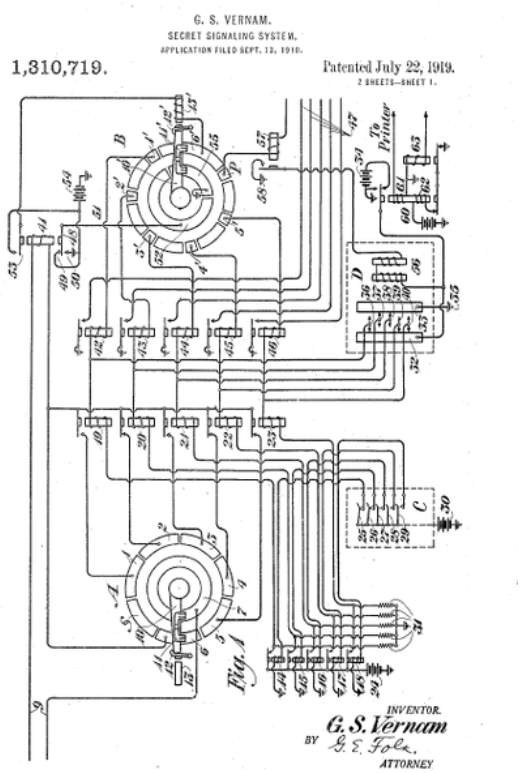
Stream Ciphers



Encrypting Long Messages

- A long message can be broken into blocks and encrypted
- Plaintext: m_i with $|m_i| = n$, where n is the block length (in bits)
- Ciphertext: c_i with $|c_i| = m$, where $m \geq n$, however, generally output size is equal to input size: $m = n$
- If $m < n$, there will be more than one ciphertext for a given plaintext, this implies ambiguity in decryption
- If $m > n$, some ciphertexts will never appear
- Encryption and decryption of a single block:

$$E_k(m_i) = c_i \quad \text{and} \quad D_k(c_i) = m_i$$

- k is the key which is used for every block

Stream Ciphers

- Plaintext: m_i with $|m_i| = k$, where k is the plaintext length (in bits), which is generally a small number: 1, 2, 4, 8, etc
- Ciphertext: c_i with $|c_i| = k$, in other words, $|m_i| = |c_i|$
- Running key: r_i with $|c_i| = k$, a sequence of symbols length k
- Plaintext, ciphertext, and running keys are from the same alphabet; for example, for $k = 4$ this would be $\{0000, 0001, \ldots, 1111\}$
- Encryption and decryption functions:

$$E(m_i) = c_i = m_i \oplus r_i \quad \text{and} \quad D(c_i) = m_i = c_i \oplus^{-1} r_i$$

where $\oplus$ is the (appropriate) addition function

A Stream Cipher — à la Vigenère

- Plaintext, Ciphertext, Running Key Alphabet: $\{a, b, c, \ldots, z\}$ encoded as elements of $\mathcal{Z}_{26}$
- Given a plaintext message: $m_i \in \mathcal{Z}_{26}$ for $i = 1, 2, 3, \ldots$
- Given a sequence of running keys: $r_i \in \mathcal{Z}_{26}$ for $i = 1, 2, 3, \ldots$
- The ciphertext sequence is computed using the encryption function

$$c_i = m_i + r_i \pmod{26}$$

- Similarly, the plaintext is computed using the decryption function

$$m_i = c_i - r_i \pmod{26}$$

Vigenère Stream Cipher

- The encryption and decryption function are

$$\begin{aligned} c_i &= m_i \oplus r_i \equiv m_i + r_i \pmod{26} \\ m_i &= c_i \oplus^{-1} r_i \equiv c_i - r_i \pmod{26} \end{aligned}$$

- The sequence of running keys r_i needs to have certain properties in order for a stream cipher to be cryptographically strong
- For the classic Vigènere:
  - The running key sequence is repeating:
    `herbalistherbalistherbalistherbali`$\cdots$
  - The period is equal to the length of the key word
  - The length of the key is generally a small integer

Cryptanalyzing Stream Ciphers

- In order to understand what properties the running key sequence needs to have we need to see if the stream cipher can be cryptanalyzed under the usual attack scenarios: CO, KP, CP, CT
- Under the CO scenario, given the ciphertext sequence c_i, the purpose of the adversary is to guess or to compute:
  - A portion or all of the running key sequence r_i
  - A portion or all of the plaintext sequence m_i
- These actions produce equivalent results in the sense that:
  - If a portion of r_i is obtained, we compute m_i using $m_i = c_i \oplus^{-1} r_i$
  - If a portion of m_i is obtained, we compute r_i using $r_i = c_i \oplus^{-1} m_i$

Cryptanalyzing Stream Ciphers

- On the other hand, under the known or chosen text attack scenarios, the adversary obtains (or chooses) a part of the plaintext sequence m_i
- This immediately implies that the adversary can compute a portion of the running key sequence r_i (which is of the same length as m_i) using

$$r_i = c_i \oplus^{-1} m_i$$

- In order to obtain longer portions of the plaintext, we cannot assume that the adversary will receive further known (or chosen) text
- At this stage, the adversary can try guess what the other (past or future) portions of the running key would be, given a portion of the running key

Properties of Running Key Sequences

- The sequence of running keys r_i needs to have certain properties in order for a stream cipher to be cryptographically strong
- Considering the CO attack scenario: The running key sequence needs to have **uniformly distributed** or **statistically random** finite segments so that all segments appear with equal probability, and any segment of the sequence cannot be guessed with better probability than the probability of that segment appearing in the sequence: **Requirement R1**
- Considering the CT attack scenario: *Given any finite segment(s) of the running key sequence*, any past or future segments need to be **unpredictable** which means they cannot be computed or guessed with better probability than the probability of that segment appearing in the sequence: **Requirement R2**

Binary Stream Cipher

- For the rest of our discussions, we will consider the binary stream cipher in which the plaintext m_i, ciphertext c_i, and the running key r_i words are binary bits, $m_i, c_i, r_i \in \{0, 1\}$ — The plaintext, ciphertext, and running key sequences are binary bit streams
- The encryption and decryption functions are the same:

$$\begin{aligned} c_i &= m_i \oplus r_i = m_i + r_i \pmod 2 \\ m_i &= c_i \oplus r_i = c_i + r_i \pmod 2 \end{aligned}$$

- The operation $\oplus$ is the mod 2 addition, which is its own inverse

m_i	0101	0010	1101	1001	0011
r_i	0110	0101	0110	0110	0101
c_i	0011	0111	1011	1111	0110

Running Key Sequence Generators

- A running key sequence generator needs to work in both sides of the communication channel, and produce exactly the same sequence r_i in order for the stream cipher to function properly

  Sender:
  m_i is created
  r_i is generated
  $c_i = m_i \oplus r_i$ is computed
  c_i is sent

  Receiver:
  c_i is received
  The same r_i is generated
  $m_i = c_i \oplus r_i$ is computed

Running Key Sequence Generators: DRNGs

- Therefore, we need to have a **deterministic state machine** generating the running key sequence
- Furthermore, in order for it to be computable, the state machine needs to be finite, i.e., it needs to have a **finite number of states**, i.e., finite memory
- A stream cipher running key generator is a deterministic finite state machine whose sequences r_i satisfy Requirements R1 and R2
- Another name for such machines: **Deterministic Random Number Generators** (DRNGs)

Random Number Generators (RNGs)

- A random number generator (RNG) produces a sequence of random (or random-looking) numbers in a predetermined range, such as $r_i \in \{0, 1\}$ or $r_i \in [0, 1]$
- Random (or random-looking) numbers have many applications: statistical physics, simulation, industrial testing and labeling, games, gambling, Monte Carlo methods, and cryptography

Deterministic vs True Random Number Generators

- There are two basic categories of RNGs:
  - True random number generators (TRNGs)
  - Deterministic random number generator (DRNGs)
- Deterministic RNGs are also known as *pseudorandom* number generators (PRNGs)
- DRNGs are algorithmic and mathematical constructs
- However, true random numbers cannot be computed on deterministic computers
- TRNGs are based classical or quantum physics processes

True Random Number Generators (TRNGs)

- True random numbers are best produced using physical random number generators which operate by measuring a well controlled and specially prepared random physical process
- Such physical processes include free running oscillators, electrical noise from a resistor or semiconductor, quantum phenomena involving photon generators and detectors
- TRNGs cannot be used as stream ciphers, except for the special case of the Vernam cipher, also called the **one-time pad**

Stream Ciphers and DRNGs

- In order to design and analyze stream ciphers, we need to study and understand the properties of DRNGs
- DRNGs are finite state machines that have a fixed but large number of starting conditions and states, and thus, very long periods
- Having long periods is an essential quality for stream ciphers; repeated sequences of running keys will yield information about the plaintext
- In addition to long period, we also would like to have DRNGs that satisfy Requirement R1 (uniform distribution) and Requirement R2 (unpredictability)

Study of DRNGs

- There are several classes of DRNGs, suitable for cryptographic and non-cryptographic applications
- The DRNGs that are particularly useful to be used for cryptography are linear and non-linear feedback shift registers
- Other types of DRNGs, such as linear congruential generators and cellular automata, can be used to generate deterministic random numbers for statistical physics, simulation, testing, games, gambling, and Monte Carlo methods
- There are also DRNGs based on number-theoretical cryptographic algorithms and elliptic curve cryptography