

Homework Assignment 3:

1. Give the value of `term` at each iteration for the following sum:

$$\frac{5\pi^5}{1536} = \frac{1}{1^5} - \frac{1}{3^5} + \frac{1}{5^5} - \frac{1}{7^5} + \dots$$

2. It is known that the inverse of Euler's constant is approximated as by computing the following sum for a large integer n :

$$\frac{1}{e} \approx 1 - \frac{1}{1!} + \frac{1}{2!} - \frac{1}{3!} + \frac{1}{4!} - \frac{1}{5!} + \dots + \frac{1}{n!}$$

We are interested in computing the sum for a given n using an iterative Python function. Starting from `sum = 0`, at each iteration, we add each `term` value to `sum`. Give the expression for `term`.

3. Given the value of `term` at the i th iteration in the above, give an efficient method `term` for the next iteration.
4. Give the value of each `term` in iteration i , by using the previous `term` for computing $\frac{2}{\pi}$ using Vieta's formula:

$$\frac{2}{\pi} = \frac{\sqrt{2}}{2} \cdot \frac{\sqrt{2+\sqrt{2}}}{2} \cdot \frac{\sqrt{2+\sqrt{2+\sqrt{2}}}}{2} \cdot \frac{\sqrt{2+\sqrt{2+\sqrt{2+\sqrt{2}}}}}{2} \dots$$

5. A product formula for computing $\pi/4$ is given by Euler:

$$\frac{\pi}{4} = \frac{3}{4} \cdot \frac{5}{4} \cdot \frac{7}{8} \cdot \frac{11}{12} \cdot \frac{13}{12} \cdot \frac{17}{16} \cdot \frac{19}{20} \cdot \frac{23}{24} \cdot \frac{29}{28} \cdot \frac{31}{32} \dots$$

where the numerators are prime numbers (starting from 3) and each denominator is the multiple of 4 nearest to the numerator.

Is there an iterative algorithm for computing this product? Explain, what the difficulty would be.
