Homework Assignment 3:

1. Describe the steps of the algorithm and give the value of term at each iteration for the following sum:

\[
\frac{5\pi^5}{1536} = \frac{1}{1^5} - \frac{1}{3^5} + \frac{1}{5^5} - \frac{1}{7^5} + \cdots
\]

2. It is known that the inverse of Euler’s constant is approximated as by computing the following sum for a large integer \( n \):

\[
\frac{1}{e} \approx 1 - \frac{1}{1!} + \frac{1}{2!} - \frac{1}{3!} + \frac{1}{4!} - \frac{1}{5!} + \cdots + \frac{1}{n!}
\]

We are interested in computing the sum for a given \( n \) using an iterative Python function. Starting from \( \text{sum} = 0 \), at each iteration, we add each term value to \( \text{sum} \). Give the expression for term.

3. Given the value of term at the \( i \)th iteration in the above, give an efficient method term for the next iteration.

4. Describe the steps of the algorithm for computing \( \frac{2}{\pi} \) using Vieta’s formula, and give the value of each term in iteration \( i \), by using the previous term:

\[
\frac{2}{\pi} = \frac{\sqrt{2}}{2} \cdot \frac{\sqrt{2 + \sqrt{2}}}{2} \cdot \frac{\sqrt{2 + \sqrt{2 + \sqrt{2}}}}{2} \cdot \frac{\sqrt{2 + \sqrt{2 + \sqrt{2 + \sqrt{2}}}}}{2} \cdots
\]

5. The Wallis formula was a product expression for computing \( \frac{\pi}{4} \). Another product formula was given by Euler:

\[
\frac{\pi}{4} = \frac{3}{4} \cdot \frac{5}{4} \cdot \frac{7}{8} \cdot \frac{11}{12} \cdot \frac{13}{12} \cdot \frac{17}{16} \cdot \frac{19}{20} \cdot \frac{23}{24} \cdot \frac{29}{28} \cdot \frac{31}{32} \cdots
\]

where the numerators are prime numbers (starting from 3) and each denominator is the multiple of 4 nearest to the numerator.

Can you give an iterative algorithm for computing this product? Explain, what the difficulties are.

Homework Assignment 3 is due 5pm on Friday Aug 24